Polytechnic
UNIVERSITY

Improving a Utility Function for Wireless Data

Virgilio Rodriguez, David J. Goodman
ECE Department, Polytechnic University

December 2002

WICAT TR 02-009

WIRELESS INTERNET
Center for Advanced Technology

Improving a Utility Function for Wireless Data*

Virgilio Rodriguez, David J. Goodman
{virgilio, dgoodman}@poly.edu
ECE Department
Polytechnic University
5 Metrotech Ctr.
Brooklyn, New York 11201

July 03, 2002

Abstract

We propose a significant improvement to a quality-of-service index on which several recently published scholarly works have been based. They use this index as a "utility function" in a formulation of radio resource management issues, in particular power control, from the perspective and explicitly using the language of microeconomic theory. The literature's index is based on an ad-hoc, difficult to interpret transformation of a physically-significant, intuitively appealing but technically misbehaving ratio. Our index is physically significant, and intuitively appealing, without the technical misbehavior which made the original ratio undesirable.

1 Introduction

Several recent scholarly publications, following an approach suggested by Ji, [3], recognize that algorithms useful for engineering applications can be obtained via the formulation of radio resource management issues, in particular power control, on the foundations of microeconomic theory. (A reader not already familiar with this formulation may benefit from consulting [1], for a general discussion) This approach is centered around the notion of a quality-of-service (QoS) index, referred to as a "utility function", defined as a real-valued function of certain physically-significant quantities. Algorithms are designed seeking to maximize, under appropriate rules and constraints, the "utility" of each transmitter.

The utility function introduced by Shah et al. [7] has found widespread acceptance in the scientific literature [2, 4, 5, 6].

However, the above function is not perfect. It is derived from the throughput-to-power ratio (TPR). Originally introduced by Zorzi and Rao [8], the TPR is a physically significant measure in bits per Joule, which is intuitively appealing as an index to guide a transmitter's energy expenditure decisions. Regrettably, this ratio grows without limit as the transmitter's power approaches zero. As a remedy, researchers made

*Supported in part by the NSF through the grant "Multimodal Collaboration Across Wired and Wireless Networks", and by the NYSTAR through the Wireless Internet CAT.

their utility function proportional to a new ratio based on an "efficiency function" obtained through an ad-hoc transformation. Unfortunately, the chosen efficiency function, while solving the technical problem at the origin, introduces problems of its own: (a) it has no clear physical interpretation, and (b) it may lead to algorithms with suboptimal performance.

Below we discuss in greater detail the TPR, the technical problem associated with the use of this ratio as a utility function, the ad-hoc transformation used by previous works to eliminate that technicality, and the drawbacks of their remedy. Subsequently, we propose an alternative remedy which eliminates the technical problem associated with the TPR, while emulating its physical significance, and rescuing and refining the original intuition. Finally, we interpret and discuss our proposal and its merits.

2 A Utility Function Favored by Previous Works

2.1 The Intuitive Index and Its Problem

Shah's utility function, [7], is derived from the ratio of a user's throughput to the power employed by the user, a quantity which had been previously used by Zorzi and Rao, [8], in an analysis of re-transmission schemes of data packets. This throughput-to-power ratio (TPR) is a physically significant quantity of considerable appeal as a user's quality-of-service index. But, a utility function proportional to this ratio would have the undesirable property of approaching infinity, as the user's power approaches zero, which is an unacceptable behavior for a "utility function".

Specifically, suppose that there is a "frame success" function, $f_s(\gamma)$, which gives the probability that a packet sent by a transmitter is successfully received, as a function of γ, the SINR (the ratio of the power in its signal to the interference-plus-noise power, both measured at the base station). The TPR is proportional to the quantity $f_s(\gamma)/P$, where P is the transmission power of the concerned transmitter. Generally, $f_s(0) > 0$, which implies that the TPR grows without bound as the transmission power approaches zero.

In particular, under suitable assumptions (see [1] for further details), the TPR takes the special form:

$$K \frac{(1 - \mathrm{BER}(\gamma_i))^M}{P_i} \tag{1}$$

In this expression, BER denotes bit error rate, P_i is the ith transmitter's power level, γ_i is the signal-to-interference-and-noise ratio for this transmitter's signal at the base station, M is the total length in bits (including overhead) of its packets, and K is a proportionality constant which yields a measure in bits per Joule. Because $\mathrm{BER}(\gamma_i)$ approaches $1/2$ as γ_i approaches zero, the TPR approaches infinity as P_i goes to zero.

2.2 The Efficiency Function Remedy

Although the quantity $f_s(0)$ is generally quite "small" for practical purposes, a misbehaving utility function inhibited the derivation of a theory of wireless resource management for data applications based on microeconomics. Hence, [7] and the literature that followed it replaced the frame success function in the numerator of the TPR with what they termed an "efficiency function", $f_e(\gamma)$. By this they meant a function whose range is the interval [0,1] which gives (as a function of the signal-to-noise ratio in a signal received at the base station) a "measure of the efficiency of the transmission protocol", [7]. Then, they defined the user's "utility function" as proportional to the ratio:

$$f_e(\gamma_i)/P_i \tag{2}$$

(Notice that, in their terminology, "efficiency function" refers to the numerator in the above ratio (2), whereas "utility function" refers to the suitably scaled ratio)

In particular, under the assumptions for which the TPR takes the form given in equation (1), they defined their efficiency function as $(1 - 2\,\mathrm{BER}(\gamma_i))^M$. This results in a utility function proportional to the ratio :

$$\frac{(1 - 2\,\mathrm{BER}(\gamma_i))^M}{P_i} \tag{3}$$

It can be verified that, for BER functions of practical interest, equation (**??**) goes to zero as the transmitter's power level goes to zero, which takes care of the technical problem under discussion.

2.3 The Problems with the Efficiency Function

Unfortunately, there is no clear physical or probability interpretation of the efficiency function, nor for the "utility function" obtained from it. Moreover, power control algorithms designed with this efficiency function can be highly suboptimal.

Discussing in detail previous works based on the efficiency function is beyond the scope of this correspondence. Nevertheless, we shall present a numerical example based on previous works to illustrate the magnitude of the potential discrepancy between a solution obtained through the efficiency function, and one based on the original frame-success function.

As discussed in [1], the utility-maximizing algorithms aim for a target signal-to-noise ratio γ^*, which is a property of the efficiency function $f_e(\gamma)$ (see equation (2)). Specifically, γ^*must satisfy

$$\gamma^* f_e'(\gamma^*) = f_e(\gamma^*) \tag{4}$$

In the numerical example in [1], $\mathrm{BER}(\gamma) = (1/2)\exp(-\gamma/2)$, and $M = 80$.

If $f_e(\gamma) = [1 - 2\,\mathrm{BER}(\gamma)]^M$, it can be verified that $\gamma_e^* = 12.4$ satisfies (4).

However, if one replaces $f_e(\gamma)$ with the original frame-success function, $f_s(\gamma) = [1 - \mathrm{BER}(\gamma)]^M$, then $\gamma_s^* = 10.75$ is the solution to $\gamma^* f_s'(\gamma^*) = f_s(\gamma^*)$.

If $\gamma^*$is feasible, and all links operate at γ^*, all of the signals arrive at the base station with the same power, Q^*, given by (see equation (20) in[1]) :

$$Q^*(\gamma^*) = \frac{\gamma^* \sigma^2}{G - (N-1)\gamma^*}$$

Above, G is the processing gain (ratio of the available bandwidth to the transmission rate) which in this particular example equals 100, and the number of terminals sharing the channel is $N = 9$.

Hence, to know which of γ_e^* or γ_s^* results in a better design, we divide the TPR resulting when $\gamma^* = \gamma_s^*$ by the TPR obtained when $\gamma^* = \gamma_e^*$, which results in the quotient:

$$\frac{f_s(\gamma_s^*)}{Q^*(\gamma_s^*)} \div \frac{f_s(\gamma_e^*)}{Q^*(\gamma_e^*)} = \frac{0.83}{0.77} \div \frac{0.92}{15.5} = 18.2$$

Therefore, the design obtained with the efficiency function yields a TPR that is inferior by a factor of 18 to the design obtained with the original frame success function.

3 Improving on the Utility Function Favored by Previous Works

3.1 Throughput: Earned vs. Serendipitous

In order to address the technicality under discussion, while preserving the physical meaning and probability interpretation of the relevant quantities, we distinguish between two additive components of the throughput: the earned (non-trivial) throughput, and the serendipitous (trivial) throughput. The earned throughput is the result of the expenditure of transmission power. On the other hand, the serendipitous throughput is that obtained without power expenditure, due to serendipity (a detector's wild guesses), which yields a correct detection of a packet with a probability of 2^{-M}.

3.2 A Refined QoS index

Now, we re-define the "utility function" to be proportional to the ratio of the *earned* throughput derived by a transmitter to the power used by this transmitter, or the earned-throughput-to-power ratio (ETPR).

Specifically, if $f_s(\gamma_i)$ gives the probability that a packet sent by transmitter i is correctly detected, when its SINR at the base station is γ_i, then we define the "utility" of user i to be proportional to the ratio:

$$\frac{f_s(\gamma_i) - f_s(0)}{P_i} \tag{5}$$

It is a simple exercise to show, via L'Hospital rule, that as long as the derivative of f_s goes to zero as γ_i goes to zero, which is the case for most interesting frame-success functions, the above ratio does go to zero with P_i.

If one wishes to make the range of the numerator equal to the interval $[0, 1]$, which may be numerically desirable, one can divide the ETPR by $(1 - f_s(0))$ (assuming $f_s(0) < 1$).

Likewise, by multiplying, as in the original index, by :

$$\frac{L}{M} R_i$$

where M is the total packet length in bits (including overhead), L is the number of information bits in the packet, and R_i is the data rate of the ith transmitter, one obtains a physically meaningful QoS index in bits per Joule.

4 Discussion and Conclusion

In most, if not all, practical systems, the serendipitous throughput is negligible, and so is the difference between the earned-throughput-to-power ratio (ETPR) and the (total) throughput-to-power ratio (TPR). Thus, the algorithm designs and performance estimates obtained on the basis of one of these ratios are indistinguishable from their homologues obtained on the basis of the other ratio. However, the ETPR is well-behaved throughout its entire domain, for which it is amenable to mathematical analysis.

Nevertheless, intellectual curiosity may lead one to consider which one of these two ratios, regardless of the technical issue at the origin, come closest to an 'ideal' QoS index. The TPR divides the average amount of data successfully transmitted (per time unit) by the energy spent (in each time unit). This yields a measure in bits per Joule which is intuitively appealing as a guide for energy-expenditure decisions. On the other hand, the ETPR compares the amount of energy spent (in each time unit), to the average amount of data (per time unit) the transmitter *would not have delivered* without energy expenditure. Hence, the ETPR is, in fact, a refinement of the intuition leading originally to the TPR.

In conclusion, the *earned*-throughput-to-power ratio (ETPR) is a legitimate QoS index for the situation of interest. This ratio exhibits good mathematical behavior, is physically significant, is defined for arbitrary frame-success functions of practical interest, and attains or surpasses the intuitive appeal of related measures already accepted by the scientific literature.

References

[1] Goodman, D., N. Mandayam, “Power control for wireless data Goodman”, IEEE Personal Communications , Volume: 7 Issue: 2 , April 2000 Page(s): 48 -54

[2] Goodman, D., N. Mandayam, "Network Assisted Power control for Wireless Data", Mobile Networks and Applications , Volume: 6 Issue: 5 , September 2001 Page(s): 409 -415

[3] Ji, H., "Resource Management in Communication Networks via Economic Models", Ph.D. Thesis, Rutgers University, New Jersey, USA, 1997

[4] MacKenzie, A.B. and S.B. Wicker, “Game theory and the design of self-configuring, adaptive wireless networks”, IEEE Communications Magazine , Volume: 39 Issue: 11 , Nov. 2001 Page(s): 126 -131

[5] Saraydar, C.U., N.B. Mandayam and D.J. Goodman ,“Pricing and power control in a multicell wireless data network”, Selected Areas in Communications, IEEE Journal on , Volume: 19 Issue: 10 , Oct. 2001 Page(s): 1883 -1892

[6] Saraydar, C.U.,N.B. Mandayam and D.J. Goodman , “Efficient power control via pricing in wireless data networks”,Communications, IEEE Transactions on , Volume: 50 Issue: 2 , Feb.2002 Page(s): 291 -303

[7] Shah, V., Mandayam, N.B. and Goodman, D.J., “Personal, Power control for wireless data based on utility and pricing”, The Ninth IEEE International Symposium on Indoor and Mobile Radio Communications, Volume: 3 , pp. 1427 -1432, 1998

[8] Zorzi, M. and R.R. Rao, “Error control and energy consumption in communications for nomadic computing” IEEE Transactions on Computers, Volume: 46 Issue: 3 , March 1997 pp: 279 -289